RADIOTRONICS

BULLETIN No. 113 **June, 1941**

CATHODE BYPASSING

Derivation of Mathematical Formula

In the Radiotron Designer's Handbook (chapter 4, page 27) a formula is given for the response of a single stage due to the effect of the cathode bypass condenser. The derivation of this formula in the more convenient, explicit form is now given and the result shown to be identical with that previously obtained. For convenience, a family of curves is also given for response against frequency, with capacitance as parameter, for type 6J7-G valves operated under standard conditions.

Analysis

The notation to be used is as follows:

- e_i = Instant. a-c. input voltage.
- e_g = Instant. a-c. grid voltage with respect to cathode.
- e_k = Instant. a-c voltage developed across self-bias resistor with respect to cathode.
- e_p = Instant. plate voltage with respect to cathode.
- e_o = Instant. *effective* output voltage.
- R_K = Resistance of self-bias resistor.
- C_K = Capacitance of cathode bypass condenser.
- R_L = *Effective* a-c plate load resistance.
- μ = Valve amplification factor *at the operating point.*
- g_m = Valve mutual conductance *at the operating point.*
- r_p = Valve plate resistance *at the operating point.*
- ω = 2πf rad./sec.
- f = Frequency of operation (c/s.).
- M = Stage amplification with self-bias resistor *completely* bypassed.
- M'' = Stage amplification with self-bias resistor *incompletely* bypassed.

In Fig. 1 the circuit is shown of a conventional self-biased stage, together with the instantaneous A.C. voltages developed across the valve electrodes and associated circuits, when an instantaneous a-c input voltage e_i is applied. The plate load impedance is assumed to be *non-reactive*

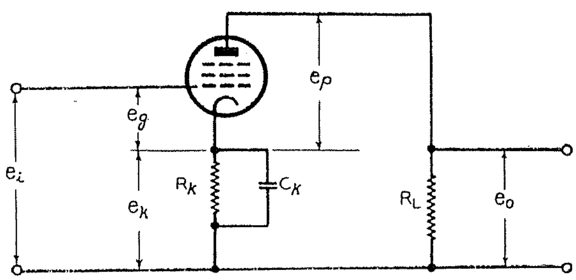


Fig. 1.

and the relative phases of all voltages referred to that of the cathode. It is then evident that

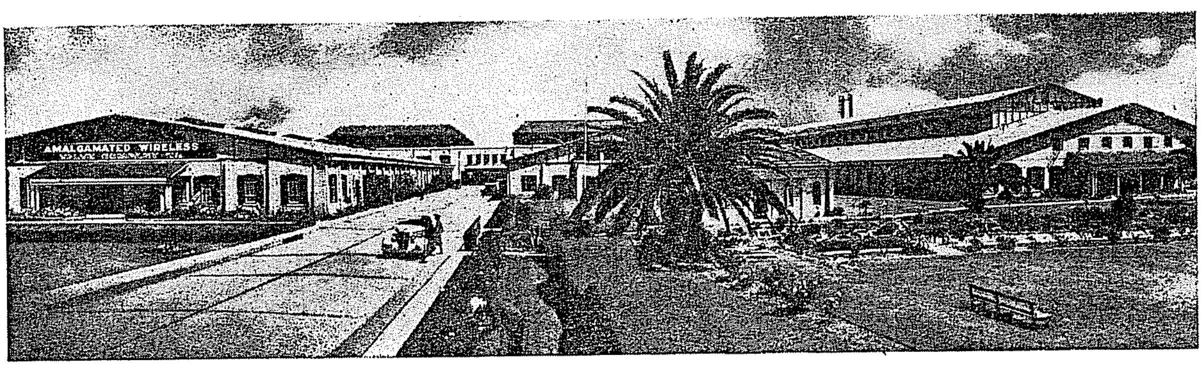
$$e_p = e_o + e_k \quad \ldots\ldots\ (1)$$

$$e_i = e_g - e_k \quad \ldots\ldots\ (2)$$

$$\text{and } e_k = e_o \cdot \frac{Z_k}{R_L} \quad \ldots\ldots\ (3)$$

where Z_k is the parallel impedance of R_K and C_K at the frequency ω rad./sec.

(Continued overleaf, column 1.)



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(Continued from page 39.)

Since $$e_p = -\mu . e_g . \frac{R_L + Z_k}{R_L + Z_k + r_p} \quad \ldots\ldots (4)$$

$$\therefore \; e_o = -\mu . e_g . \frac{R_L}{R_L + Z_k + r_p} \quad \ldots\ldots (5)$$

Also, $$e_i = e_g . \left(1 + \frac{\mu . Z_k}{R_L + Z_k + r_p}\right) \ldots (6)$$

The *degenerative* stage amplification M'' with the impedance Z_k in the cathode circuit is then given by—

$$M'' = \frac{e_o}{e_i} = \frac{-\mu R_L}{R_L + r_p + Z_k(\mu + 1)} \ldots (7)$$

When $Z_k \to 0$ at the frequency ω rad./sec., i.e. when the self-bias resistor R_K is *completely bypassed at this frequency*, the *normal* stage amplification M is given by—

$$M = \frac{e_o}{e_g} = \frac{-\mu R_L}{R_L + r_p} \quad \ldots\ldots\ldots\ldots (8)$$

$$(Z_k \to 0).$$

Hence the ratio to which the stage amplification is reduced *at the frequency* ω *rad./sec.* due to incomplete cathode bypassing is—

$$\frac{M''}{M} = \frac{1}{1 + \frac{\mu + 1}{R_L + r_p} . Z_k} \quad \ldots\ldots\ldots (9)$$

Substituting for Z_k

$$Z_k = \frac{R_k}{1 + j\omega C_k R_k} \quad \ldots\ldots\ldots\ldots (10)$$

and putting

$$\frac{\mu + 1}{R_L + r_p} = \gamma \quad \ldots\ldots\ldots\ldots\ldots (11)$$

the expression (9) becomes

$$\frac{M''}{M} = \frac{1}{1 + \gamma . \frac{R_k}{1 + j\omega C_k R_k}} \quad \ldots\ldots (12)$$

of which the *modulus* is

$$\left|\frac{M''}{M}\right| = \sqrt{\frac{1 + (\omega C_k R_k)^2}{(1 + \gamma R_k)^2 + (\omega C_k R_k)^2}} \ldots (13)$$

which gives *explicitly* the *exact scalar ratio* to which the voltage amplification of a stage is reduced at the frequency ω rad./sec. due to incomplete cathode bypassing.

The *attenuation* in decibels is then—
Attenuation—

$$(db) = 20 \log \sqrt{\frac{1 + (\omega C_k R_k)^2}{(1 + \gamma R_k)^2 + (\omega C_k R_k)^2}} \ldots (14)$$

In the Radiotron Designer's Handbook (Chap. 4, page 27) the formula given is

$$\left|\frac{M''}{M}\right| = \sqrt{\frac{1 + (\omega C_k R_k)^2}{(1 + aMR_k)^2 + (\omega C_k R_k)^2}} \ldots (15)$$

where $$a = \frac{1}{M(R_L + r_p)} + \frac{1}{R_L} \quad \ldots\ldots (16)$$

and the other symbols are as defined in the present notation. This formula is in an implicit form but on substituting the expression (8) for M in the first term in the denominator, it is seen that—

$$aM = \frac{\mu + 1}{R_L + r_p} = \gamma$$

and the two formulae become *identical.*

In the case of *high-mu pentode valves*, generally

$$r_p >> R_L$$
$$\& \; \mu >> 1$$
$$\text{so that } \gamma \doteqdot g_m \ldots\ldots\ldots$$

and the *exact* expression (13) may be written as the more immediately applicable *approximation*—

$$\left|\frac{M''}{M}\right| = \sqrt{\frac{1 + (\omega C_k R_k)^2}{(1 + g_m R_k)^2 + (\omega C_k R_k)^2}} \quad \ldots (17)$$

which is usually sufficiently accurate for most practical purposes.

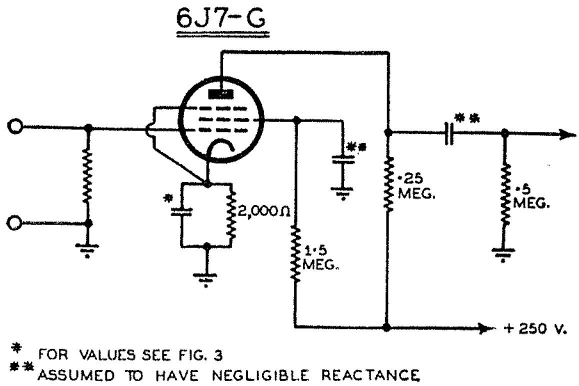

Fig. 2.

Owing to the number of independent variables involved, it is difficult to represent the formulae (13) or (16) in convenient, general graphical form. A useful set of curves, however, is obtained by plotting a family of curves of C_k for $\left|\frac{M''}{M}\right|$ or 20 log $\left|\frac{M''}{M}\right|$ against f for relevant valve types, when operated under specified standard conditions.

In Fig. 3, the latter family of curves has been plotted for type 6J7-G (6C6, 1603) when operation is under the following conditions:

E_b = 250 volts
R_L (DC) = 0·25 megohm
R_G = 0·5 megohm
R_S = 1·5 megohms
R_K = 2,000 ohms
g_m = 750 μmhos
r_p = 4·0 megohms approx.

The *effective* A.C. plate load resistance R_L is given by

$$R_L = \frac{R_L(D.C.) \times R_g}{R_L(D.C.) + R_g}$$

$$= \frac{\cdot 25 \times \cdot 5}{\cdot 25 + \cdot 5} = \cdot 167 \text{ Megohm.}$$

$$\text{and } \gamma = \frac{\mu + 1}{R_L + r_p} = \frac{g_m \,.\, r_p + 1}{R_L + r_p}$$

$$= \frac{(750 \times 10^{-6})\,(4 \times 10^{6}) + 1}{\cdot 167 + 4}\ \mu\text{mhos}$$

$$= \frac{3001}{4 \cdot 167}\ \mu\text{mhos}$$

$$= 720\ \mu\text{mhos}$$

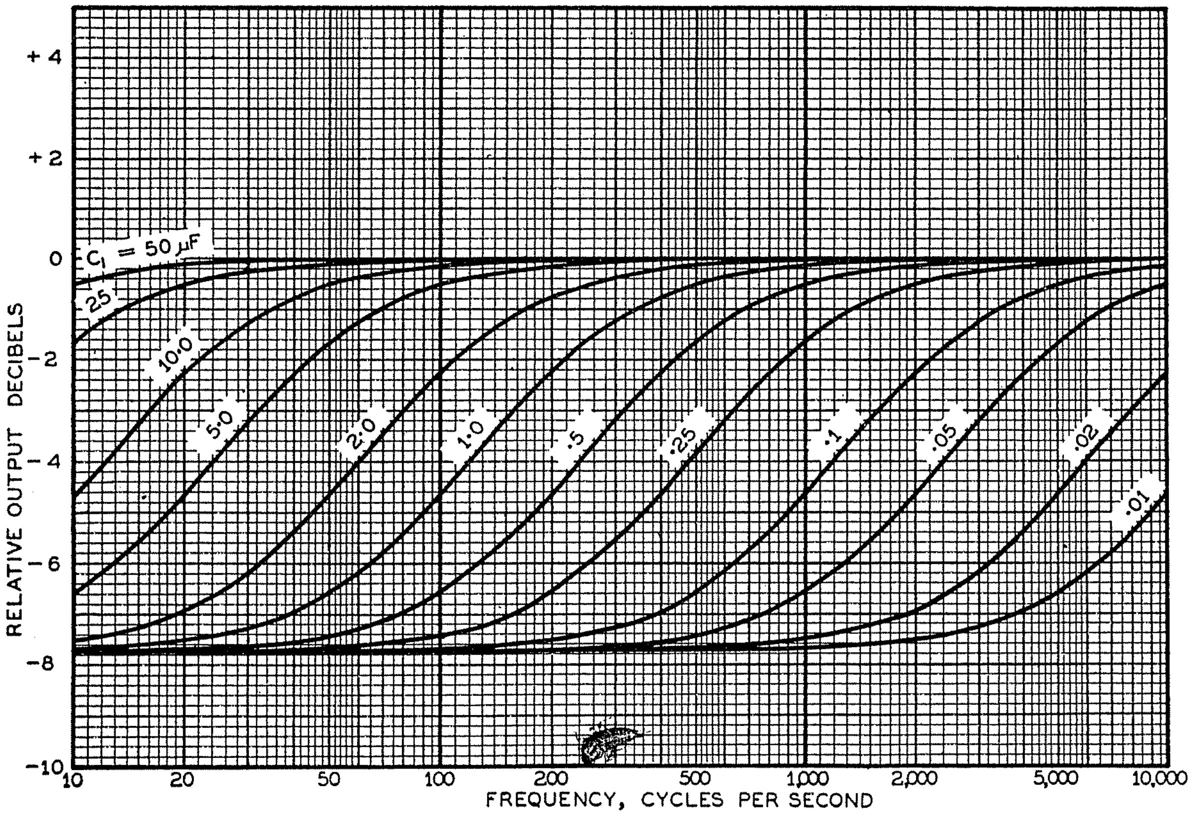

Fig. 3.